Disease With Vaccine

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MATH 1550-01

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Overview

The Problem

- A 1978 British Medical Journal article reported about a flu epidemic at a boys' boarding school. On January 22nd of that year, only one boy had the flu, which none of the other boys had ever had. By the end of the epidemic on February 4th that year, 512 of the 763 boys in the school had contracted the disease. Suppose that 0.218% of the total number of possible contacts results in the disease being spread from one child to another. Suppose also that a boy was usually sick with this flu for 2 days.
- Adjust the SIR model to allow for vaccination of susceptible boys. Assume that 15% are vaccinated each day and that immunization begins after 3 days. Discuss the effect on the duration and intensity of the epidemic. Consider the impact of other vaccination rates. Discuss your results.

Our Assumptions

- With the given information we were able to assume that initially, there were 762 boys susceptible, 1 boy infected, and 0 boys in recovery.
- We can also assume that the virus takes its typical course for only 3 days before the vaccinations begin. The spread of the virus will then slow down because there will be less susceptible boys.
- The transmission probability (a) is given to us: 0.00218
- The average people that get sick in one week (b) can easily be determined by 1/2 (the length of the disease): 0.5

Before the Vaccine

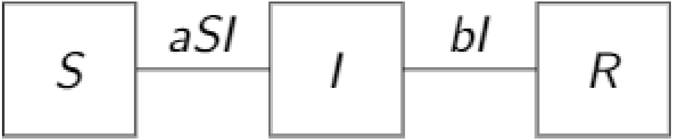


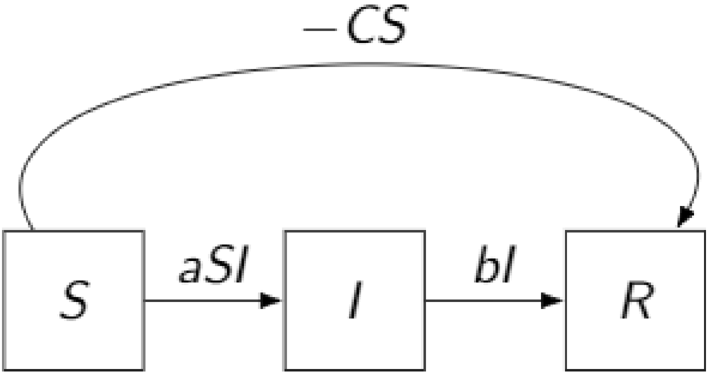
Figure: SIR Model

Before the Vaccine

The equations in a difference equation:

$$S_{n+1} = S_n - .00218 S_n I_n \quad S_0 = 762$$
$$I_{n+1} = I_n - .5 I_n + .00218 S_n I_n \quad I_0 = 1$$
$$R_{n+1} = R_n + .5 I_n \quad R_0 = 0$$

With the Vaccine



With the Vaccine

The equations as a differential equation (implemented after the first 3 days):

$$\frac{dS}{dt} = .00218SI - .15S$$
$$\frac{dI}{dt} = .00218SI - .5I$$
$$\frac{dR}{dt} = .5I + .15S$$

C is .15 because 15% of the boys are vaccinated each day after the first three days.

Python Code and Graphs

```
import matplotlib.pyplot as plt
import numpy as np

n=763
h=0.1
length=2

a=.00218
b=1/length
vaccine=.15

start=0
end=14
```

To begin the code, we have to import some libraries, and then we begin declaring the variables we are going to use.

Python Code and Graphs

```
dSdt=lambda I,S,v: (-a*I*S)-(v*S)
dIdt=lambda I,S: (-b*I)+(a*I*S)
dRdt=lambda I,S,v: (b*I)+(v*S)

time=np.linspace(start, end,((end-start)/h)+1)
I=np.zeros (len(time))
S=np.zeros (len(time))
R=np.zeros (len(time))

I[0]=1
S[0]=n-I[0]
R[0]=0
```

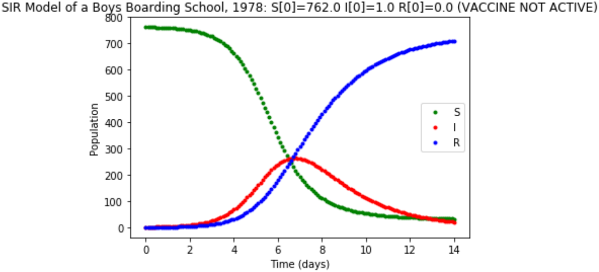
Continuing, we start with the basic differential equations for each section of the population (S, I, and R). We create 4 arrays, named time, S, I, and R, all with the same length. We then initialize the first index for each, the population at the start of the epidemic.

Python Code and Graphs

```
BeforeVac=(3.0/h)+1
for j in range (1 , len(time)):
    if j<BeforeVac:
        S[j]=S[j-1]+h*dSdt(I[j-1], S[j-1],0)
        I[j]=I[j-1]+h*dIdt(I[j-1], S[j-1])
        R[j]=R[j-1]+h*dRdt(I[j-1],S[j-1],0)
        print(str(round(time[j],2)) + "\t" + str(round(S[j],2))+"\t"+ str(round(I[j],2))+ "\t"+str(round(R[j],2)))
    else:
        S[j]=S[j-1]+h*dSdt(I[j-1], S[j-1],vaccine)
        I[j]=I[j-1]+h*dIdt(I[j-1], S[j-1])
        R[j]=R[j-1]+h*dRdt(I[j-1],S[j-1],vaccine)
        print(str(round(time[j],2)) + "\t" + str(round(S[j],2))+"\t"+ str(round(I[j],2))+ "\t"+str(round(R[j],2)))
```

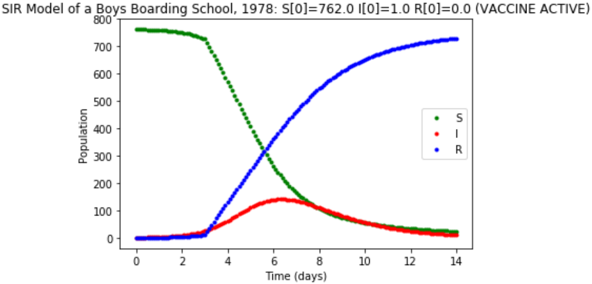
This section includes some of the most important code in doing the calculations for the SIR model. Using Euler's method, estimations to the differential equations are solved, from the start to end of the epidemic. It also takes into account the use of the vaccine, with administration beginning after 3 days.

Python Code and Graphs



This is a graph of our results without a vaccine being used on the population.

Python Code and Graphs



This is a graph of our results with a vaccine being used on the population

Conclusion

Note the differences between the previous graphs, and the results we obtain from using vs. not using a vaccine. In the case of using a vaccine, the peak number infected occurs sooner, and with a lot less people. Also when using a vaccine, the susceptible population has a sudden drop, and the recovered has a sudden gain. On the other hand, not using the vaccine causes a gradual decrease and increase for those populations.